Econ 101A
Section 14

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1 Long-Run Equilibrium in Competitive Markets

- In the short-run, the number of suppliers is fixed.
- In the long-run though, the number of suppliers can change. In particular, whenever there are positive profits, more firms will enter the market, since they stand to make a positive profit.
- Entering firms will affect the market supply in several ways (for prices above average cost):
  - More will be produced, or $Y^S(p)$ increases for all p above average cost. Graphically, this shifts the supply curve rightward (which means that the equilibrium price and profits will decrease)
  - The supply curve will be more elastic, or $\frac{\partial Y^S(p)}{\partial p}$ will increase, for all p above average cost. Graphically, this flattens the supply curve.
- Additional firms will enter the market until profits are zero, or until price equals average cost.

2 Welfare

2.1 Introduction

- **Welfare** is a measure of how well off an agent (either consumer or producer) is. We use this to compare the state of the agent before and after a change in the economy to see if they are better or worse off.
- As an aside: recall the Fundamental Theorem of Calculus (FToC): for a differentiable function f, we have the following:

$$f(b) = f(a) + \int_a^b f'(x)dx$$

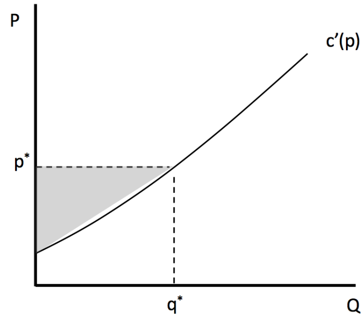
2.2 Producer Surplus

- **Producer surplus** is simply firm's profits - a measure of how much revenue is higher than the cost for the producer. For price of output good p and output y_0, this is defined as follows:

$$\begin{aligned} PS &= \pi(y_0) = py_0 - c(y_0) \\ \text{(by FToC)} \quad &= p * 0 - c(0) + \int_0^{y_0} (p - c'(y))dy \\ &= \int_0^{y_0} (p - c'(y))dy - c(0) \end{aligned}$$

- If $c(0) = 0$, the $c(0)$ term drops out in the above expression. Note that you can graph this in two ways on "price/quantity" axes:
  1. The rectangle bordered by the price axis, the market price, the profit-maximizing quantity, and the average cost (we knew this from class, $\pi = (P - AC)y_0$).

2. The "triangular region" bordered by the price axis, the price, and the marginal cost curve (we learn this from the integral notation we derived above) - this has been graphed below.



- If we wanted to compare the producer's well-being at two different prices, simply compare their producer surpluses in both states of the world (i.e., compare their profits).

2.3 Consumer Surplus

- **Consumer surplus** measures how much less one is spending than their willingness to pay.
- A *change in* consumer surplus meausres how much more or less one must spend under a *new* price in order to reach the utility level achieved under *old* prices.
- That is, the *change in* consumer surplus is the difference in a consumer's expenditure, or **compensating variation**, due to a price change.[1]
- Formally, suppose u was the utility under p_0, and the price changes to p_1. We can write our change in consumer surplus in terms of our expenditure function as follows:[2]

$$\begin{aligned} e(p_0, u) - e(p_1, u) &= \int_{p_1}^{p_0} \frac{\partial e(p,u)}{\partial p} dp \\ \text{(by Shepard's lemma)} \qquad &= \int_{p_1}^{p_0} h^*(p,u) dp \end{aligned}$$

(Note in lecture slides, we omit $*$ in $h^*(p,u)$ for notational simplicity. Recall Shepard's lemma

$$\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} = \frac{\partial [p_1 h_1 + p_2 h_2 - \lambda(u(h_1, h_2) - \bar{u})]}{\partial p_i}\bigg|_{h_i = h_i^*} = h_i^*(\mathbf{p}, \bar{u})$$

which is the result we get by applying the envelope theorem to expenditure function.)

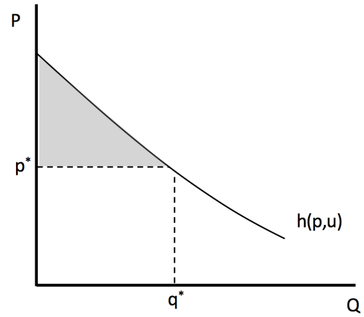
- The consumer surplus is formally defined as the limit of the above expression when your initial price $p_0 \to \infty$. In other words:

$$CS = \int_{p_1}^{\infty} h^*(p,u) dp$$

- Therefore, we can graph the consumer surplus as the "triangular region" bounded by the market price, the *Hicksian* demand curve, and the price axis. This has been graphed below.

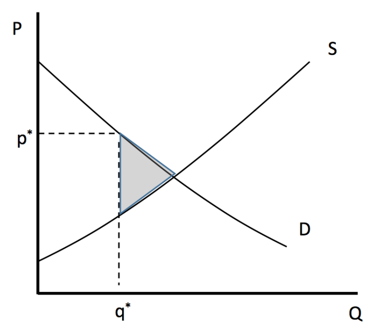
[1] At first glance, you may think comparing the utilities would be a more natural method to measure the consumer's well-being. However, you may recall that the actual numerical value for utility doesn't have any meaning, since we can scale utility functions and still represent the same preferences, so this would not be very meaningful.

[2] You may think it's odd that we are subtracting the "final" expenditure *from* the "initial" expenditure. It makes more sense to write it this way though, if we want more consumer surplus to indicate the consumer is better off. For instance, if $p_1 > p_0$, we would want consumer surplus to be negative, which will be true, as written.



2.4 Deadweight Loss

- Total welfare is the sum of economic surplus of all agents in the economy.
- For example, in the case of tax, total welfare = consumer surplus + producer surplus + tax revenue.
- **Deadweight loss** is the loss in total welfare due to a distortion in the market in comparison to the efficient market outcome. Examples of distortions include:
  - taxes and subsidies
  - price floors or ceilings (e.g. rent control)
  - non-competitive markets (e.g. monopoly market structure)
- Graphically, deadweight loss is the area bounded by the "triangular region" bounded by the supply curve, the demand curve, and the (non-competitive) market quantity sold. This has been graphed below.



3 Exercises

3.1 Profit Maximization with Taxes (Problem 2, Midterm 2, Spring 2012)

We consider the market for widgets, which is characterized by the aggregate (inverse) demand function $p(X) = a - bX$, where X is the total quantity of widgets demanded in the market. The cost function of each company is $c(y) = cy^{\alpha}$, with $c > 0$.

1. Assume perfect competition (that is, the price p of the widget is given) and set up the profit maximization of each firm.
2. Solve for the profit-maximizing level of production $y^*(p)$ (that is, the supply function) using the first-order condition.
3. Check the second-order conditions. Under what values of the parameters are they satisfied? Interpret the economic significance of this parameter restriction.

4. Now consider the conditions for the market equilibrium. For points 4-7, assume that the parameters are such that the second order conditions are satisfied. Assume that N firms produce and write the equation for the equilibrium price p^* that equates aggregate supply and demand. Do not attempt to solve explicitly for p^*.

5. Now introduce taxation. Denote by p the price inclusive of tax that the consumer pays, and by $p - t$ the price net of tax that accrues to the producer. Rewrite the market equilibrium condition.

6. Use the implicit function theorem to compute $\partial p^*/\partial t$.

7. Show that $0 < \partial p^*/\partial t < 1$. What does it mean economically?

3.2 Consumer Surplus (Problem 2, Midterm 2, Spring 2009)

We evaluate here the change in consumer surplus associated with a change in the price of good 1 from p_1 to p_1' for a consumer with Cobb-Douglas utility $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, with $0 < \alpha < 1$.

1. Explain the intuition of why the change in consumer surplus is defined as $\Delta CS = e(p_1, p_2, u) - e(p_1', p_2, u)$, where e is the expenditure function. (6 points)

2. Define the expenditure function. (4 points)

3. Derive an expression for the expenditure function $e(p_1, p_2, u)$ for this Cobb-Douglas case given that the Hicksian demands in this case are

$$\begin{aligned} h_1^*(p_1, p_2, u) &= u \cdot (\frac{p_2}{p_1} \cdot \frac{\alpha}{1-\alpha})^{1-\alpha} \\ h_2^*(p_1, p_2, u) &= u \cdot (\frac{p_1}{p_2} \cdot \frac{1-\alpha}{\alpha})^{\alpha} \end{aligned}$$

   (5 points)

4. Solve for the change in consumer surplus $\Delta CS = e(p_1, p_2, u) - e(p_1', p_2, u)$. (If you get stuck here, just skip to the next point) (5 points)

5. If you also substituted for u the expression for the indirect utility $v(p_1, p_2, M)$ you would get that ΔCS is proportional to

$$\left[1 - \left(\frac{p_1'}{p_1}\right)^\alpha\right] \cdot M \tag{1}$$

   (Do not attempt to do this, it involves a fair amount of algebra, take it as given) Using expression (1), show that the following statements are true, and provide intuitive explanations for each of them: (i) $\Delta CS > 0$ if and only if $p_1' < p_1$; (ii) holding constant p_1' and p_1 with $p_1' < p_1$, ΔCS is increasing in α; (iii) holding constant p_1' and p_1 with $p_1' < p_1$, ΔCS is increasing in M. (15 points)