

Econ 101A

Section 17

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1 General Equilibrium

We're going to take the case with two agents A and B and two goods, 1 and 2. We will assume the agents have utility functions $u_A(x_1^A, x_2^A)$ and $u_B(x_1^B, x_2^B)$, respectively, and have initial endowments (ω_1^A, ω_2^A) and (ω_1^B, ω_2^B) , respectively. There is no other production technology of goods 1 and 2.

- In a barter economy (exchange economy), there are no prices.
- In a Walrasian economy, we add prices to goods. The price of good 1 is p_1 and the price of good 2 is p_2 .

We can represent the 2-agents and 2-goods economy using an **Edgeworth box**.

1.1 Contract Curve (for barter economy)

In an Edgeworth box diagram, the **contract curve** represents a set of possible allocations in a barter economy (when you simply trade for the goods). A given allocation¹ $((x_1^A, x_2^A), (x_1^B, x_2^B))$ is on the contract curve if it satisfies the following two properties:

- **Individually rational (=Pareto improvement)**, meaning that this allocation provides at least as much utility to each agent as the endowment does. Formally, the allocation is individually rational if for $i = A, B$, we have

$$u_i(x_1^i, x_2^i) \geq u_i(\omega_1^i, \omega_2^i)$$

Graphically, this is the oval-shaped region bounded by the two agents' indifference curves that pass through the endowment.

- **Pareto efficient**, meaning that you cannot move to another allocation and make both agents better off (or you cannot make one agent better off without making the other worse off). Formally, the allocation is Pareto efficient if there is no other allocation $((\hat{x}_1^{A*}, \hat{x}_2^{A*}), (\hat{x}_1^{B*}, \hat{x}_2^{B*}))$ in this economy such that the following holds for $i = A, B$, and the inequality is strict for at least one player:

$$u_i(\hat{x}_1^{i*}, \hat{x}_2^{i*}) \geq u_i(x_1^{i*}, x_2^{i*})$$

¹Definition: An assignment of total endowments to agents such that $x_j^i \geq 0$ for all i and j , $x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$, and $x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$.

Visually, this is the set (line) of points where the agents' indifference curves are tangent to one another. Assuming strongly monotonic preferences, this so-called **Pareto set** necessarily connects the two origins, or the lower left and upper right hand corners of the Edgeworth box. Note that the Pareto Set is independent of the initial endowment.

1.2 Finding Walrasian Equilibrium

In the Walrasian set-up, we add prices to our goods. $((x_1^{A*}, x_2^{A*}), (x_1^{B*}, x_2^{B*}), p_1^*, p_2^*)$ is a Walrasian equilibrium (i.e., competitive equilibrium) if each individual maximizes and market clears. That is,

$$(x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i(x_1^i, x_2^i) \text{ s.t. } p_1^* x_1^i + p_2^* x_2^i \leq p_1^* w_1^i + p_2^* w_2^i \text{ for } i = A, B$$

and

$$x_j^A + x_j^B \leq w_j^A + w_j^B \text{ for all } j = 1, 2$$

The key to finding the Walrasian equilibrium is to find the equilibrium price (ratios).

1.2.1 Graphical Solution: Offer Curves

- Agents i 's **offer curve** is the consumption pairs of points (x_1^{i*}, x_2^{i*}) that solve the aforementioned utility maximization problem for given prices, p_1, p_2 .
- Visually, this is the set of points that are tangent to any possible indifference curve of i and any budget line that is a downward sloping line (with slope $-p_1/p_2$) that passes through the endowment.
- To accurately graph offer curves, it may help to combine your equations such that x_2^i is a function of x_1^i . In other words, solve for p_1/p_2 in your demand function for x_1^{i*} and then substitute this expression in for the price ratio in your demand function for x_2^{i*} . See Exercise 2.1 as an example.
- Important features of the offer curve:
 - Each agent's offer curve passes through the endowment.
 - Every intersection of the agents' offer curves except for the original endowment is a Walrasian equilibrium.²

1.2.2 Analytical Solution

- **Step 1:** Solve a standard utility maximization problem with the only twist being that income M is now the value of the endowment. The solution to this standard utility maximization problem gives us consumer demands $x_1^{i*}(p_1, p_2, \omega_1^i, \omega_2^i), x_2^{i*}(p_1, p_2, \omega_1^i, \omega_2^i)$ for $i = A, B$. Note that prices are *relative*, meaning we are really only need to solve for the price ratio p_1/p_2 .
- **Step 2:** Plug your demand functions $x_j^{A*}(p_1, p_2), x_j^{B*}(p_1, p_2)$ for one good into the market clearing condition: $x_j^{A*}(p_1, p_2) + x_j^{B*}(p_1, p_2) = \omega_j^{A*} + \omega_j^{B*}$, for all $j = 1, 2$.

²If the endowment is Pareto efficient, it will also be a Walrasian equilibrium, but in general, it will not be.

Make sure you understand exactly what this condition means. How does it relate to the crossing of offer curves?

- **Step 3:** Solve for p_2/p_1 in the constraint above (only the ratio of prices is identified), this is your equilibrium price ratio. (Alternatively, you could just fix one price to be 1 and then solve explicitly for the other price.) Plug the prices into $x_j^{A*}(p_1, p_2)$, $x_j^{B*}(p_1, p_2)$ for each good j to obtain the equilibrium quantities/allocation.

1.3 Important Theorems

- **First Welfare Theorem:** All Walrasian Equilibria are on the Contract Curve (and therefore, are Pareto Efficient).
 - In other words, “the market” will naturally achieve an allocation that will leave no additional improvements available through trade.
 - Note this theorem says nothing about equity of the equilibrium, it just says the equilibrium will be efficient.
 - Note: this result may fail when there are market distortions (e.g. taxes, price controls, asymmetric information, etc.)
- **Second Welfare Theorem:** If preferences are convex, then for every Pareto efficient allocation $((x_1^A, x_2^A), (x_1^B, x_2^B))$ there exists some endowment point (ω_1, ω_2) such that $((x_1^A, x_2^A), (x_1^B, x_2^B))$ is a Walrasian Equilibrium for endowment (ω_1, ω_2) .
 - In other words, social planners can redistribute endowments to achieve any Pareto efficient allocation they desire.
 - In particular, social planners can reallocate endowments to achieve more equitable allocations.

2 Exercises

2.1 Simple General Equilibrium Example

Consider an exchange economy in which each agent's utility function is: $u_i(x_1^i, x_2^i) = x_1^i * x_2^i$ for $i = A, B$. Initial endowments are $\omega^A = (2, 8)$ and $\omega^B = (8, 2)$. We're going to find the competitive (Walrasian) equilibrium.

1. Write and solve the utility maximization problem for agent A.
2. Write and solve the utility maximization problem for agent B.
3. Using the above demands, plot the offer curves of each player. Solve for x^{A*} and x^{B*} .
4. Plug demands for good 1 into the market clearing condition for good 1
5. Solve for p_2^*/p_1^* and x^{A*}, x^{B*} . [By a result called *Walras' Law*, we only need to solve the market clearing condition for one of the goods to know that the markets for both goods will be in equilibrium at this price.] Does this allocation look like what we found visually in part 3?
6. Is this outcome Pareto efficient? If so, why? If not, how would we show it isn't?

2.2 Utility Maximization and Barter (Problem 1, Final, Spring 2008)

Consider consumers with preferences $u(x_1, x_2) = \min(x_1, x_2)$ over the two good x_1 and x_2 .

1. Draw the indifference curves for this utility function and provide an example of goods that could approximately fit this pattern of preferences.
2. Consider a standard utility maximization problem. Each consumer solves the problem

$$\begin{aligned} \max_{x_1, x_2} \quad & \min(x_1, x_2) \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leq M \end{aligned}$$

Solve for $x_1^*(p_1, p_2, M)$ and $x_2^*(p_1, p_2, M)$. Provide the steps for the solution. [Hint: Do not go for straight for the derivatives, help yourself with the figure in point 1]

3. Compute how the optimal solutions for $x_1^*(p_1, p_2, M)$ and $x_2^*(p_1, p_2, M)$ varies as a function of p_1 and p_2 . Provide economic intuition.
4. Now solve a problem similar to the one in point 2, except that each consumer is endowed not with income M , but with initial endowments of the two goods ω_1 and ω_2 . Hence, each consumer solves

$$\begin{aligned} \max_{x_1, x_2} \quad & \min(x_1, x_2) \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2 \end{aligned}$$

Solve for $x_1^*(p_1, p_2, \omega_1, \omega_2)$ and $x_2^*(p_1, p_2, \omega_1, \omega_2)$

5. Compute how the optimal solutions for $x_1^*(p_1, p_2, \omega_1, \omega_2)$ and $x_2^*(p_1, p_2, \omega_1, \omega_2)$ varies as a function of p_1 and p_2 . Compare to your answers in point 3. Provide economic intuition.

6. Now assume that there are two identical consumers, consumers A and B, each with preferences $u(x_1, x_2) = \min(x_1, x_2)$ and initial endowments (ω_1^A, ω_2^A) and (ω_1^B, ω_2^B) . Assume first $(\omega_1^A, \omega_2^A) = (1, 3)$ and $(\omega_1^B, \omega_2^B) = (3, 1)$. Draw the Edgeworth box for this economy and determine the Individually Rational area, that is, the area of points that each individual prefers to the initial allocation.
7. Still on the Edgeworth box, and still assuming $(\omega_1^A, \omega_2^A) = (1, 3)$ and $(\omega_1^B, \omega_2^B) = (3, 1)$, determine the Pareto Set, that is, the set of allocations $(x_1^A, x_2^A), (x_1^B, x_2^B)$ that are feasible and such that there is no other allocation $(\hat{x}_1^A, \hat{x}_2^A), (\hat{x}_1^B, \hat{x}_2^B)$ that *both* consumer would prefer to $(x_1^A, x_2^A), (x_1^B, x_2^B)$ [Notice that this does not restrict the analysis to points in the Individually Rational Area] Argue your logic on this one.
8. Using the answers in points 6 and 7, determine the Contract Curve, that is, the set of points that are possible outcomes of barter starting for $(\omega_1^A, \omega_2^A) = (1, 3)$ and $(\omega_1^B, \omega_2^B) = (3, 1)$.
9. For general (ω_1^A, ω_2^A) and (ω_1^B, ω_2^B) but assuming $\omega_1^A + \omega_1^B = \omega_2^A + \omega_2^B$ (that is, a square Edgeworth box), determine mathematically the expression for the Contract Curve in the barter case for two consumers for preferences $u(x_1, x_2) = \min(x_1, x_2)$.

2.3 More general equilibrium (Problem 3, Final, Fall 2016)

Jon ($i = 1$) and Nick ($i = 2$) are stuck on the island of Evans. There are two goods that are essential to their well-being: shorts ($j = 1$) and T-shirts ($j = 2$). Jon and Nick have different preferences over shorts and T-shirts. Jon's preferences are governed by $U_1(x_1, x_2) = x_1 x_2^2$ and Nick's preferences can be described by $U_2(x_1, x_2) = x_1^2 x_2$.

1. Draw an Edgeworth Box with the following endowments: $\omega_1 = (8, 2)$ and $\omega_2 = (2, 8)$.

Strange occurrences happen on Evans and somehow Jon ended up with all the shorts $\omega_1 = (10, 0)$ and Nick ended up with all the T-shirts $\omega_2 = (0, 10)$. Assume that these are the endowments going forward.

1. Compute the marginal rate of substitution for consumers 1 and 2.
2. Without doing any additional calculations, is the initial endowment a Walrasian equilibrium? How do you know?
3. Describe the Pareto efficient allocations in this economy (give a mathematical answer and illustrate your answer in the Edgeworth box). (5 points)
4. Write down the budget constraint for each individual.
5. What class of preferences do the agents have? What is particular about the demand derived from these preferences?
6. Solve for each individual's Marshallian demands as a function of p_1 and p_2 .
7. Solve for the market clearing price in the Walrasian equilibrium. Is this price unique? If so, explain why. If not, give an alternative price that would generate the same equilibrium.