Econ 101A
Section 19

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1 Game Theory

Let's focus on the static game for this section. A **static game** is a game where players take actions simultaneously. In the next section, we will discuss dynamic games, in which agents make decisions sequentially.

1.1 Notation

- A game consists of a set of players $(1, 2, ..., I)$, a strategy set S_i for each player, and a full specification of payoffs to each player for each possible combination of strategies.
- A 2-player game with discrete strategy sets can be represented by a matrix. This representation is known as **normal form**, and makes sense when players act simultaneously. For example:

		Player 2		
		L	C	R
	T	1,2	2,1	3,4
Player 1	M	2,1	5,0	2,6
	B	3,7	8,2	5,5

- The **strategy set** S_i is the set of possible strategies for player i. In the game represented above, $S_1 = \{T, M, B\}$ and $S_2 = \{L, C, R\}$. In oligopoly production, which cannot be represented in matrix form, the strategy set of firm i is $S_i = \{y_i \in \mathbf{R}^+\}$.
- A **strategy profile** $s = (s_1, s_2)$ is an assignment of strategies to each player. $s = (T, R)$ describes the event in which player 1 chooses strategy T and player 2 chooses strategy R. We will attempt to "solve" games by figuring out which strategy profiles are equilibria (more about this later).
- Recall that the purpose of game theory is to study situations in which a player's outcome depends on the actions of others (as well as her own). Otherwise, a utility maximization problem would be good enough!

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- In matrix form, cell (i, j) displays the **utility payoffs** if player 1 chooses strategy i and player 2 chooses strategy j. Payoffs are always listed in the order (player 1, player 2).

  $U_1(T, R) = 3$

  $U_2(B, C) = 2$

- When there are more than two players, s_{-i} represents the strategies of all players other than player i. For example, if there are 4 players, $s_{-2} = (s_1, s_3, s_4)$. "For all $s_{-i} \in S_{-i}$" means: for every possible combination of strategies for the other players in the game. (In a two-player game, this reduces to "for every strategy of the other player".)

1.2 Best Response

- A player's optimal strategy often depends on what the other players are doing. In other words, a player might have a different "best response" to each action of another player.

- A strategy s'_i is a **best response** to rivals's strategies s_{-i}, if $U_i(s'_i, s_{-i}) \geq U_i(s_i, s_{-i})$ for all $s_i \in S_i$.

- We denote the set of best responses as $BR_i(s_{-i})$.

- Notice, that this definition is weaker than the definition for a dominant strategy. What is the only difference between the two definitions? We've dropped the "for all $\forall\ s_{-i}$."

1.3 Dominant Strategies

- In some cases, a player has a strategy that is unambiguously superior (dominant strategy), regardless of what the other player chooses.

- A strategy s_i^* is a **dominant strategy** for player i if $U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i})$ for all $s_i \in S_i$ and for all $s_{-i} \in S_{-i}$.

- In words: No matter what the strategies of the other players are, player i gets a (weakly) higher utility payoff from playing s_i^* than from playing any other strategy available to him.

- Strategy profile $s^* = (s_i^*, s_{-i}^*)$ is an **equilibrium in dominant strategies** if $U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i})$ for all $s_i \in S_i$, for all $s_{-i} \in S_{-i}$ and all $i = 1, ..., I$

- In words: If each player's strategy is a dominant strategy, then the combination of these strategies is an equilibrium in dominant strategies. This is very intuitive: each player is playing the move that is always best for him or her. However, it is not always this easy to "solve" a game.

1.4 Pure Strategy Nash Equilibrium

- **Nash equilibrium** may exist in pure or mixed strategies.

- If we can find a set of strategies $s^* = (s_1^*, s_2^*, ..., s_I^*)$ such that for each player i, s_i^* is a best response to s_{-i}^*, then we've found a pure-strategy Nash equilibrium.

- Strategy profile $s^* = (s_i^*, s_{-i}^*)$ is a **pure-strategy Nash equilibrium** if $U_i\left(s_i^*, s_{-i}^*\right) \geq U_i\left(s_i, s_{-i}^*\right)$ for all $s_i \in S_i$ and $i = 1, ..., I$
- In other words, strategy profile $s^* = (s_i^*, s_{-i}^*)$ is a pure-strategy Nash equilibrium where each player's strategy is a best response to another player's strategy. That is, a Nash equilibrium is where "best response meets best response (a fixed point)."
- In other words, strategy profile $s^* = (s_i^*, s_{-i}^*)$ is a pure-strategy Nash equilibrium if no player has a unilateral profitable deviation.
- Dominant strategy equilibrium is a special case of pure-strategy Nash equilibrium: A pure-strategy Nash equilibrium where each player plays a dominant strategy.

1.5 Mixed Strategy Nash Equilibrium

- A **mixed strategy** is an assignment of non-zero probability to each strategy, e.g. $s_1 = (1/2, 1/2)$.
- In a **mixed-strategy Nash equilibrium**, at least one player plays a mixed strategy, and no player has a profitable unilateral deviation.
- In other words, in a mixed-strategy Nash equilibrium, each player mixes strategies such that the other player is indifferent between her own strategies.
- Nash theorem: Every finite game has a Nash equilibrium.
- If a game has no pure strategy Nash equilbria, then it has a mixed strategy NE.
- Almost all games have an odd number of NE, so if there are two pure strategy NE, then there is generally also a mixed strategy NE.

2 Exercises

2.1 Dominant Strategies

Applying the definition: Consider the game represented by

		Player 2		
		L	C	R
	T	1,2	2,1	3,4
Player 1	M	2,1	5,0	2,6
	B	3,7	8,2	5,5

1. Prove or disprove that $s_2 = R$ is a dominant strategy for player 2.
2. Prove or disprove that $s_1 = B$ is a dominant strategy for player 1.
3. Does the game have an equilibrium in dominant strategies (aka dominant strategy equilibrium)

2.2 Nash Equilibrium

1. What is the Nash equilibrium in the 3x3 game above?

2. Find the pure strategy Nash equilibrium or equilibria in a game of Chicken.

1\2	Veer	Don't Veer
Veer	2,2	1,3
Don't Veer	3,1	0,0

2.3 Mixed Strategy NE

1. Find the mixed strategy equilibrium for this Battle of the old GSIs.

Jon\Nick	BTS	Stary Kids
BTS	2,1	0,0
Stray Kids	0,0	1,2