Econ 101A
Section 20

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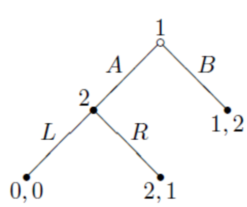
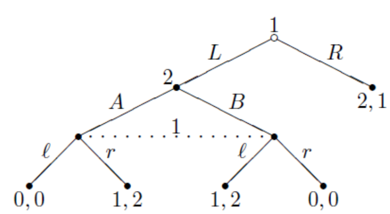
1 Dynamic Games

- Players move sequentially.
- At least some of these moves are observed by the other players before subsequent moves.
- **Extensive form:** This is the tree-like representation of a game. Typically, a dynamic game can be represented by extensive form (but static games can also be represented by extensive form).
- **Information set:** If the player knows exactly which actions have preceded her choice, then her information set is simply her node in the game (for this class, node and information set will be equivalent).
- **Strategy:** A player's strategy s_i must indicate what her action will be *at every information set*, even at information sets that will not be reached according to this strategy!
- **Examples:**





- In the game on the right, player 1's strategy will be an element in the set

$$S_1 = \{(L, l, l'), (L, l, r'), (L, r, l'), (L, r, r'), (R, l, l'), (R, l, r'), (R, r, l'), (R, r, r')\}$$

  , because player 1 has three information sets (one at the beginning of the game and two after player 2 moves). Player 2 has only one information set, then the strategy set is:

$$S_2 = \{A, B\}$$

- **Subgame:** A subgame is the tree structure formed by each node x and its successors.

- **Subgame Perfect Nash Equilibrium (SPNE):** A strategy profile is a SPNE if it specifies a Nash equilibrium in every subgame of the whole game.

- **Backwards Induction:** The process of analyzing a game from back to front (from information sets at the end of the tree to information sets at the beginning). At each information set, one insures that players play optimally given strategies in the remainder of the game. It can be verified that backward induction is a straight forward technique for finding a subgame perfect Nash equilibrium of the game.

1.1 Stackelberg Duopoly

- In the previous two models of oligopoly (Bertrand and Cournot), each firm chooses its action not knowing the other firms' actions. How do the conclusions change when the firms move sequentially? Is a firm better off moving before or after the other firms?

- In Stackelberg's game, each firm chooses an output (just like in Cournot's game), but the firms make their decisions sequentially, rather than simultaneously: one firm chooses its output, then the other firm does so, knowing the output chosen by the first firm.

- Assumptions: A single good is produced by two firms ($i = 1, 2$); each firm can produce q_i units of the good at a cost of $C_i(q_i)$ and the price at which output is sold when the total output is Q is $P_d(Q)$.

- The profit function of firm i can be written as $\pi_i(q_1, q_2) = q_i \cdot P(q_1 + q_2) - C_i(q_i)$ for $i = 1, 2$..

- Firm 1 moves at the start of the game. Thus a strategy of firm 1 is simply an output. Firm 2 moves after every history in which firm 1 chooses an output. Thus a strategy of firm 2 is a function that associates an output for firm 2 with each possible output of firm 1.

- Let's assume $C_i(q_i) = cq_i$ for $i = 1, 2$ and assume that the inverse demand function is linear and it's given by $P(Q) = \alpha - Q$ for $Q \leq \alpha$ and $P(Q) = 0$ for $Q > \alpha$, where $\alpha > 0$ and $c \geq 0$ are constants.

- The game has a finite horizon, so we may use backward induction to find its subgame perfect equilibria.

  - First, for any output of firm 1, we find the outputs of firm 2 that maximize its profit. Under the above assumptions firm 2 has a unique best response to each output q_1 of firm 1, given by

$$BR_2(q_1) = \begin{cases} \frac{1}{2}(\alpha - q_1 - c) & \textit{if } q_1 \leq \alpha - c \\ 0 & \textit{if } q_1 > \alpha - c \end{cases}$$

- Next, we find the output of firm 1 that maximizes its profit, given the strategy of firm 2.

- When firm 1 chooses the output q_1, firm 2 chooses the output $BR_2(q_1)$, resulting in a total output of $q_1 + BR_2(q_1)$, and hence a price of $(\alpha - (q_1 + BR_2(q_1)))$.
- Thus, firm 1's output in a subgame perfect equilibrium is a value of q_1 that maximizes

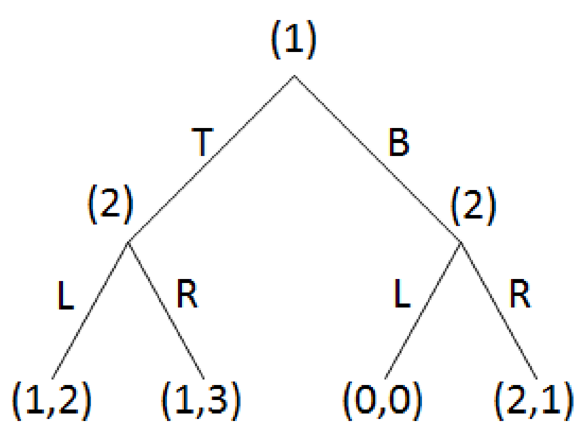
$$\pi_1(q_1, BR_2(q_1)) = q_1(\alpha - q_1 - \frac{1}{2}(\alpha - q_1 - c) - c)$$

- The FOC is given by $(\alpha - q_1 - \frac{1}{2}(\alpha - q_1 - c) - c) - \frac{1}{2}q_1 = 0 \quad \Rightarrow \quad q_1^* = \frac{\alpha - c}{2}$
- Thus, Stackelberg's game has a unique SPNE, in which firm 1's strategy is the output $q_1^* = \frac{\alpha-c}{2}$ and firm 2's strategy is given by plugging in q_1^* into its best response function, that is, $q_2^* = BR_2(q_1^*) = \frac{\alpha-c}{4}$.
- Under the above assumptions firm 1, given the ability to move first and commit to a level of production, produces more output and obtains more profit in the subgame perfect equilibrium of the sequential game in which it moves first than it does in the Nash equilibrium of Cournot's game, and firm 2 produces less output and obtains less profit.

2 Exercises

2.1 Dynamic game

Consider the game below:



1. How many subgames does it have?
2. How many pure Nash equilibria? (pure strategy profiles that are mutual best responses)
3. If this was a *simultaneous* game, how many Nash equilibrium would it have?
4. Using backward induction, find the subgame perfect Nash equilibrium of the game.

2.2 Cournot vs Stackelberg

Using the same setting from last section (Section 20). Let's consider a market for cars with two firms with identical cost function given by $C(q) = 2q$. The market inverse demand function is given by $P(Q) = 9 - Q$.

1. What will the Cournot equilibrium output for each firm be? (See last section!)
2. What is the Stackelberg equilibrium output for each firm if firm 2 enters second?
3. How much profit will each firm make in the Cournot game? How much in Stackelberg?
4. Which type of market do consumers prefer: Cournot duopoly or Stackelberg duopoly? Why? (Hint: Think about consumer surplus)

2.3 Externality and Pigouvian tax

Consider a competitive market for a good. The inverse demand function is

$$P(Q) = 30 - Q,$$

and firms have constant private marginal cost

$$MC_p(Q) = 6.$$

Production creates pollution. The marginal external damage from aggregate production is increasing and given by

$$MD(Q) = 2 + Q.$$

Assume there are many competitive firms, so that in an unregulated market firms take the price as given and produce until price equals private marginal cost.

1. Find the competitive equilibrium quantity and price in the absence of regulation.
2. Find the socially efficient quantity. Carefully explain which marginal cost curve should be equated to demand.
3. Compute the deadweight loss from the unregulated equilibrium.
4. Suppose the government can impose a per-unit tax t on production. What Pigouvian tax implements the socially efficient quantity?
5. What price do consumers pay, and what net-of-tax price do producers receive, under the optimal tax?
6. Suppose instead that the government imposes a constant tax equal to the marginal damage evaluated at the unregulated equilibrium quantity. Would this tax be too high, too low, or exactly right? Explain briefly.