Econ 101A
Section 7

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1 Intertemporal choice

In an intertemporal-choice problem, an individual is deciding how to allocate consumption across different periods, meaning she is deciding how much to spend and to save in each period. are deciding how to allocate.

There are two new "ingredients" in this problem, δ and r.

- **δ, the discount rate**, represents the individual's degree of impatience - i.e. her preference for consuming today versus consuming tomorrow. Given $\delta > 0$, she prefers receiving a given amount x today over receiving that same amount x tomorrow. This is because she values tomorrow's x at $\frac{1}{1+\delta}x$ today, and since $\delta > 0$, $\frac{1}{1+\delta} < 1$.
- **r, the interest rate**, represents the difference in cost between consuming today and consuming tomorrow. If the individual wants to consume more than M_0 in period 0, then in period 1 she needs to repay the bank $(1+r)$ times the amount you borrowed $(c_0 - M_0)$. If on the other hand the individual wants to consume less than M_0 in period 0, she can save some first-period income M_0, then in period 1 she receives an interest payment of $(1+r)$ times the amount saved $(M_0 - c_0)$.

1.1 Interpreting δ

We can write your preferences as:

$$\{x \text{ today}\} \succsim \{y \text{ tomorrow}\} \text{ if } x \geq \frac{1}{(1+\delta)}y$$
$$\{x \text{ today}\} \precsim \{y \text{ tomorrow}\} \text{ if } x \leq \frac{1}{(1+\delta)}y.$$

1.2 Utility Function

We can write the individual's utility function (at $t = 0$) as:

- For two periods: $u(x_0, x_1) = U(x_0) + \frac{1}{1+\delta}U(x_1)$
- For a total of T periods: $u(x_0, ..., x_T) = \sum_{t=0}^{t=T} \frac{1}{(1+\delta)^t}U(x_t)$.

1.3 Budget Constraint

- The budget constraint is no longer a fixed number M. Rather, the individual's consumption in period 1 depends on how much she borrowed or saved in period 0, and on the interest rate r.

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- Think of the constraint as

$$c_0 + c_1 \leq M_0 + M_1 + (M_0 - c_0)(r)$$

- $M_0 - c_0$ is the amount *saved* (if positive) or *borrowed* (if negative).

1.4 Optimization

- Therefore, for a two-period individual, her maximization problem can be written as:

$$\max_{c_0, c_1} U(c_0) + \frac{1}{1+\delta} U(c_1), \text{ s.t. } c_0 + \frac{1}{1+r} c_1 \leq M_0 + \frac{1}{1+r} M_1$$

- How to interpret δ? r?
- First-order condition:

$$U'(c_0^*) - \frac{1+r}{1+\delta} U'(M_1 + (1+r)(M_0 - c_0^*)) = 0$$

- If $\delta = r$: FOC can be simplified as $U'(c_0^*) = U'(c_1^*)$. As long as $U'(\cdot)$ is monotonic, we have $c_0^* = c_1^*$. Intuitively, consumer would perfectly **smooth consumption** across time.
- If $\delta \neq r$, we can apply Implicit Function Theorem to derive the following comparative statics:

$$\frac{\partial c_0^*}{\partial M_0} = \frac{(1+r)^2 U''(c_1^*)}{(1+\delta)U''(c_0^*) + (1+r)^2 U''(c_1^*)} > 0$$
$$\frac{\partial c_0^*}{\partial M_1} = \frac{(1+r) U''(c_1^*)}{(1+\delta)U''(c_0^*) + (1+r)^2 U''(c_1^*)} > 0$$
$$\frac{\partial c_0^*}{\partial r} = \underbrace{\frac{U'(c_1^*)}{(1+\delta)U''(c_0^*) + (1+r)^2 U''(c_1^*)}}_{\text{substitution effect, } <0} + \underbrace{\frac{(1+r) U''(c_1^*)}{(1+\delta)U''(c_0^*) + (1+r)^2 U''(c_1^*)}(M_0 - c_0^*)}_{\text{income effect, } <0 \text{ if } M_0 < c_0^*, \ >0 \text{ if } M_0 > c_0^*,}$$

2 Exercises

2.1 Labor supply

1. Write down expenditure minimization of labor supply problem.
2. Use Envelop Theorem to derive $\frac{\partial e(p, w, \overline{u})}{\partial w}$.
3. Derive Slutsky equation for labor supply $h = H - l$ at $\overline{u} = V(p, w, M)$ (Just to remind you, $V(p, w, M)$ is the indirect utility function from Marshallian demand)

2.2 Intertemporal Choice

From Section (2) above, FOC can be rearranged as the following:

$$U'(c_0^*) - \frac{1+r}{1+\delta} U'(c_1^*) = 0$$

1. Write down c_1^* as a function of c_0^*, M_0, M_1, and r, and substitute c_1^* in the equation above.
2. Use Implicit Function Theorem to derive $\frac{\partial c_0^*}{\partial M_0}$, $\frac{\partial c_0^*}{\partial M_1}$, and $\frac{\partial c_0^*}{\partial r}$

2.3 Intertemporal Choice with Linear Utility

An individual lives for two periods, $t = 0, 1$. Her utility in each period is $U(c_t) = \alpha c_t$, and her in each period is M_t. Assume that the interest rate is r and that the agent discounts the future by $\frac{1}{1+\delta}$.

1. Write down the utility-maximization problem.
2. Draw the indifference curves and the budget lines.
3. What can you say about the relation between consumption in period 0 and consumption in period 1?
4. Calculate consumption at the corner solutions and use them to prove your argument from part 3.