Econ 101A
Section 8

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1 Expected Value and Expected Utility

Recall: The *expected value* of any "lottery or probabilistic draw is the sum of all possible outcomes times their respective probabilities:

$$E(X) = \sum_i x_i * Pr\{x_i\}$$

where $\sum_i Pr\{x_i\} = 1$ and i indexes each possible value of X.

- If you are calculating expected value (wealth) then solve the formula above with w_i in the place of x_i.
- If you are calculating expected utility, then $U(w_i)$ goes in the place of x_i.

1.1 Illustration — Selling a textbook

Suppose you want to sell an used textbook. You have a friend who offers you \$40 for the book. There's an 80% chance you'd be able to sell the book for \$60 if you post a flyer on campus, but also a 20% chance it won't sell at all, and by then you will have lost the opportunity to sell it to your friend. Your initial wealth is $\hat{w}$.

1. What is the expected monetary value (the expected wealth outcome) of each option?
   (a) selling to your friend: $Ew = \hat{w} + \$40$
   (b) posting a flyer: $Ew = \hat{w} + .8\$60 + .2\$0 = \hat{w} + \$48$
2. What is the expected utility of each option, and which option would you choose, if $U(w) = \ln(w)$?
   (a) selling to your friend: $EU = \ln(\hat{w} + \$40)$
   (b) posting a flyer: $EU = .8\ln(\hat{w} + \$60) + .2\ln(\hat{w})$

   You will choose option (a) over (b) if $\ln(\hat{w} + \$40) > .8\ln(\hat{w} + \$60) + .2\ln(\hat{w})$.

1.2 Jensen's inequality

This is a useful theorem that allows us to compare the utility of a lottery (or any uncertain outcome) to the utility of a certain outcome *with the same expected value as the lottery.*

- If $f(x)$ is concave, then $f(Ex) \geq Ef(x)$.
- If $f(x)$ is strictly concave, then $f(Ex) > Ef(x)$.

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Think of $f(x)$ as a utility function $u(x)$. The theorem says that if my utility function is strictly concave, then I will always prefer to receive \$$y$ than to play a lottery with expected value \$y. For example: if you offer me 50% chance of 100 and 50% chance of 0, I would rather get \$50 for sure than play that lottery if my utility function is strictly concave. (In fact, I may prefer \$40 for sure or even \$30 for sure depending how concave/curved my utility function is... but the theorem tells us I will certainly prefer \$50 because that is the expected outcome of the lottery, and I don't like uncertainty).

1.3 Risk Preferences (a closer look)

We can interpret the agents' preferences over risk when she has utility over wealth $u(w)$ using a graph.

- Facing the lottery of selling the book leaves you with an expected utility $E[u(w)]$.
- The Jensen inequality tells us that if $u(x)$ is strictly concave then we will strictly prefer the utility of a certain outcome with the same expected value as the lottery ($u(Ew)$) to the expected utility of the lottery ($E(u(w))$).
- We can define the **certainty equivalence** (C.E. in the graph) that gives the amount of wealth that leaves you indifferent between the lottery and this certain level of wealth.
- The difference between the expected value of the lottery and the certainty equivalence of the lottery is your **risk premium**. This is the amount of expected value you are willing to give up to avoid the risk from facing the lottery. The more risk averse you are, the greater the risk premium is (the farther left the C.E. moves).

1.4 Measures of Risk Aversion

- Coefficient of absolute risk aversion: $r_A = -\frac{U''(w)}{U'(w)}$
- Coefficient of relative risk aversion: $r_R = r_A w = -\frac{U''(w)}{U'(w)} w$
- Both measures are positive for concave (and increasing) utility functions and negative for convex (and increasing) utility functions.
- Larger positive numbers indicate stronger risk-aversion, i.e. you'd be willing to give up *more* expected wealth in exchange for less uncertainty.

2 Application

2.1 Insurance

- There is a possibility that a bad event will happen in the future (for example, I may crash my car). If I am **risk averse** (concave utility function), we know that my expected utility will be *less* than the utility of my expected wealth.
- As a result, I would rather smooth my income over each possible state of the world. I need to decide how much money to transfer from my "good states" to my "bad states". Insurance is one way to do this. For insurance:
  - You are paid the **coverage** α when things go wrong (like when you crash your car)
  - You pay the insurance company your **premium** $q\alpha$ no matter what happens
- To be explicit: suppose the following:
  - You have w dollars as your wealth
  - There is a probability p that you will have an accident which makes you lose L dollars

- In this case, if we were asked to choose our desired coverage, we would solve the following maximization problem (assuming $\alpha \geq 0$)

$$\max_{\alpha}(1-p)u(w-q\alpha)+pu(w-q\alpha-L+\alpha)$$

- Note that we say an insurance scheme is **fair** if the individual is expected to lose the same amount of money whether or not he/she has insurance.
- Note that if an individual is risk averse, insurance companies may charge higher than the fair price; that is, you will be expected to pay more when you're insured than when you are not.

2.2 Investment

- Basic problem: you want to determine how much of your wealth to invest in stock, which can sometimes yield high returns and sometimes yield low returns.
- To explicitly outline the model:
  - α is the share of your wealth w that you invest in stocks. We want to choose our optimal α
  - r_+ is the (high) rate of return (on stocks) that happens with probability p r_- is the low rate of return (on stocks) that happens with probability $1-p$
- Our maximization problem is as follows:

$$\max_{0\leq\alpha\leq1}(1-p)u(w[(1-\alpha)+\alpha(1+r_-)])+pu(w[(1-\alpha)+\alpha(1+r_+)])$$

- Note you can apply this to any set up where you're putting money in a risky investment (like gambling, for example).

3 Exercises

3.1 Problem 1, MT2 Fall 2002

In the world, we observe many individuals who purchase insurance and gamble, a puzzling behavior. Define the problem as follows. An agent has utility function $u(w)$ with $u' > 0$. The agent has wealth w.

1. Consider the following stylized Las Vegas gamble: the agent wins \$10 with probability 1/10 and loses \$2 with probability 9/10. Write the expected value and expected utility associated with this gamble.
2. Try to show that a risk-averse agent (concave utility, $u'' < 0$) will prefer not to take this gamble using Jensen's inequality. Risk-averse people do not go to Las Vegas.

3.2 Problem 2, MT2 Fall 2004

Mary is worried about car accidents next year. She has \$10,000 in wealth, including the value of her car. With probability 2/3 she will have an accident and suffer a loss of \$7,500 (she will not be hurt, just the car). With probability 1/3 she will not have an accident, and her losses will be \$0.

1. What is Mary's expected wealth?
2. Assume $u(w) = w^{1/2}$, where w is the wealth left over after the accident. What is her expected utility?
3. Mary can purchase insurance by paying a premium of \$5,100. This insurance will fully reimburse the damage (\$7,500) in the case of an accident, and will give no payment in the case of no accident. What is Mary's expected wealth if she takes the insurance? Is the insurance premium fair?
4. Will Mary take up the insurance? (Compute the expected utility and compare to expected utility in point 2)

5. Angela is a friend of Mary. She hears Mary talk about her decision and exclaims "*I* would not have purchased the insurance! Give an example of a utility function such that Angela would *not* have purchased the insurance given the same wealth and accident probabilities as Mary.

6. Angela adds: "Mary, you are so risk-averse, relax! Let's leave aside the 'relaxing' issue. Provide intuition on why risk-aversion translates into a concave utility function, like $u(w) = w^{1/2}$.