Econ 101A

Section 13

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1 Aggregation and Market Equilibrium

- So far, we have learned to solve a firm's supply function of a good (quantity produced as a function of price) and a consumer's demand function for a good (which also depends on the good's price).
- We can add up the supply functions of different firms to obtain an **aggregate supply** function; similarly, we can add up the consumers' demand functions to obtain an **aggregate demand** function.
- The **equilibrium price** of good i ($i = 1, ..., n$) is the value of p_i such that aggregate supply of i equals aggregate demand of i, or:

$$Y_i^S(p_i^*, w, r) = X_i^D(p_1^*, ..., p_n^*, M^1, ..., M^J).$$

- The steps for finding equilibrium price and quantity are as follows (note that if $n = 1$, meaning that you just have 1 good, you do not need to worry about the subscript i):
  1. Solve each firm's supply function for each good i.
  2. Add up all the firms' supply functions for good i to get $Y_i^S(p_i, w, r)$ for each i. (If the supply function is the same for all firms, then simply multiply it by the number of firms).
  3. Solve for each consumer's demand function.
  4. Add up all the consumers' demand functions for each good i to get $X_i^D(p_1, ..., p_n, M^1, ..., M^J)$ for all i. (Usually each demand function will depend on each consumer's income M^j, so we will end up with these individual-specific terms in our aggregate demand function).
  5. Set the aggregate supply function (found in Step 2) equal to the aggregate demand function (found in step 4), and solve for $\mathbf{p}^* = (p_1^*, ..., p_n^*)$. These are your equilibrium prices.
  6. Plug the p_i^*s into either $Y_i^S(p_i, w, r)$ or $X_i^D(p_1, ..., p_n, M^1, ..., M^J)$ to get the **equilibrium quantity** of good i.

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2 Elasticities

- Elasticities show how one variable changes (in percentage terms) in response to a change (in percentage terms) in another variable.
- General expression: $\epsilon_{x,y} = \frac{\partial x}{\partial y}\frac{y}{x}$.
  We interpret this as *the percent change in x as a response to a percent change in y.*
- You have already learned to calculate changes such as dx^*/dp either directly or through the Implicit Function Theorem. By expressing these changes as elasticities, you can make these measures "unit-free" An elasticity relates a *percent* change in one variable to a *percent* change in another.
- Example: price elasticity of (own) demand: $\epsilon_{x^*,p} = \frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*}$

3 Exercises

3.1 Production with Fixed Costs

Suppose a firm has a cost function $c(y) = H + y^2$ if $y > 0$ and $c(0) = 0$.

1. Derive and draw the MC and AC curves.
2. Solve for y at the point where the two curves intersect, and show that this point is also the minimum of the AC curve.
3. Draw the supply function for the firm with cost function $c(y) = H + y^2$ if $y > 0$ and $c(0) = 0$, and write an expression representing it: $(y^*(p))$.
4. If p stays constant, how does y^* respond to an increase in H?
5. What is the profit function of this firm?

3.2 Market Equilibrium

Consider an economy of 3 individuals and 2 firms producing one good.

- Individual 1 has demand function: $x_1 = 500 - p$
- Individual 2 has demand function: $x_2 = 750 - p$
- Individual 3 has demand function: $x_3 = 250 - p$
- Firm 1 has supply function: $Y_1 = 300 + 4p$
- Firm 2 has supply function: $Y_2 = 700 + 2p$

Answer the following questions:

1. What is the aggregate demand?
2. What is the aggregate supply?
3. What is the equilibrium price?
4. What is the equilibrium quantity sold in the market?

3.3 Demand, Supply, and Taxes (Problem 1, MT2, Spring 2012)

Assume the case of perfect competition, with each firm i producing quantity q_i with total cost $C_i(q_i) = cq_i$. That is, costs are linear in the quantity produced.

1. Determine the marginal cost function C'_q and the average cost function $C(q)/q$, and plot the two functions in a graph with x-axis quantity q_i and y-axis cost/price.

2. Plot graphically the supply function for each firm. Also, write it analytically, in the form $q^* = S(p)$, that is, what the quantity supplied q^* is as a function of p.

3. Assume now that in perfect competition there are 5 firms, all with the same cost function $C_i(q_i) = cq_i$. How does the aggregate supply function differ from the individual supply function, if at all? Plot, and write analytically.

4. Assume now that aggregate demand is given by the linear (inverse) demand function $p(Q) = A - bQ$, with $A > c$. Draw in the graph with the marginal cost function of point (3). If you want, assume $A = 10$, $c = 5$, $b = 1$. Solve (graphically or otherwise) for the equilibrium perfect competition price p^*_{PC}, as well as for the overall quantity produced Q^*, and obtain also analytical solutions. Notice that the equilibrium will be where demand equates supply. How do quantity and price depend on A and c?