# Econ 101A
# Section 14

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Two topics today that we have not practiced yet. The first is game theory: what happens when my best choice depends on what you do. The second is choice under risk: how a person who dislikes risk values a gamble, and how much insurance she buys. Both come down to one or two simple conditions. Learn to write them down quickly.

## 1 Games

**The pieces.** Players $i = 1, \ldots, I$. Each player picks a strategy $s_i$ from a set $S_i$. Player $i$'s payoff $U_i(s_i, s_{-i})$ depends on her own strategy and on everyone else's, $s_{-i}$. In a payoff matrix, Player 1 picks the row, Player 2 the column, and the first number is Player 1's payoff.

**Dominant strategy.** $s_i^*$ is dominant if it is a best choice *whatever the others do*: $U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i})$ for all $s_i$ and all $s_{-i}$. If every player has one, that is an equilibrium in dominant strategies.

**Nash equilibrium.** Each player's strategy is a best response to what the others actually play: $U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)$ for all $s_i$ and all $i$. No one wants to change alone. A dominant-strategy equilibrium is always a Nash equilibrium. The reverse is false.

**Finding pure-strategy equilibria.** For each column, mark Player 1's best row. For each row, mark Player 2's best column. A cell with both marks is a Nash equilibrium.

**Mixed strategies.** A player can randomize. If Player 1 is willing to mix between two rows, both rows must give her the same expected payoff. So the condition that pins down *Player*

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*2's* probability is *Player 1's* indifference, and the other way around. This feels backwards at first. It is the most common mistake on exams.

# 2 Risk

**Expected utility.** A lottery gives wealth $c_1$ with probability $\pi$ and $c_2$ with probability $1-\pi$. The person ranks lotteries by $\pi u(c_1) + (1-\pi)u(c_2)$.

**Attitude to risk.** Compare a lottery with getting its expected value for sure. If $u$ is concave ($u'' < 0$), the sure thing is better: risk averse. If $u$ is linear, risk neutral. If $u$ is convex, risk loving.

**Measuring risk aversion.** The curvature of $u$ alone is not meaningful, since $u$ can be rescaled. The Arrow-Pratt measures divide by $u'$:

$$r_A(c) = -\frac{u''(c)}{u'(c)}, \qquad r_R(c) = -\frac{c\,u''(c)}{u'(c)} = c\,r_A(c).$$

$r_A$ is about gambles of a fixed size in dollars. $r_R$ is about gambles of a fixed share of wealth.

**Certainty equivalent and risk premium.** The certainty equivalent CE is the sure amount the person likes as much as the lottery: $u(\text{CE}) = E[u(c)]$. The risk premium is $E[c] - \text{CE}$: what she would give up, on average, to get rid of the risk.

# 3 Insurance

A person has wealth $W$ and loses $L$ with probability $\pi$. She can buy $\alpha$ units of coverage. Each unit costs $q$, paid in both states, and pays 1 if the loss happens. Final wealth:

$$c_{\text{loss}} = W - L + (1-q)\alpha, \qquad c_{\text{no loss}} = W - q\alpha.$$

She maximizes $\pi\,u(c_{\text{loss}}) + (1-\pi)\,u(c_{\text{no loss}})$. The first-order condition for an interior $\alpha$ is

$$\pi(1-q)\,u'(c_{\text{loss}}) = (1-\pi)\,q\,u'(c_{\text{no loss}}).$$

Left: what one more unit of coverage gives her, in the bad state. Right: what it costs her, in the good state.

**The key result.** If insurance is *actuarially fair*, $q = \pi$, the condition becomes $u'(c_{\text{loss}}) = u'(c_{\text{no loss}})$. With $u$ strictly concave this means $c_{\text{loss}} = c_{\text{no loss}}$: full insurance, $\alpha = L$. If $q > \pi$, each unit costs more than it pays on average, and a risk-averse person buys only partial insurance, or none.

# 4 Exercises

**Exercise 1 (dominant strategies)**

(a) Two coffee shops choose whether to Advertise ($A$) or Not ($N$):

| | $A$ | $N$ |
|---|---|---|
| $A$ | $(3,3)$ | $(8,1)$ |
| $N$ | $(1,8)$ | $(6,6)$ |

Does each player have a dominant strategy? Find the Nash equilibrium. Is it the outcome that maximizes the sum of payoffs?

(b) True or false: every Nash equilibrium is an equilibrium in dominant strategies.

**Exercise 2 (a coordination game)** Two hunters choose to hunt a Stag ($S$) or a Hare ($H$). A stag needs both of them. A hare can be caught alone.

| | $S$ | $H$ |
|---|---|---|
| $S$ | $(5,5)$ | $(0,3)$ |
| $H$ | $(3,0)$ | $(3,3)$ |

(a) Does either player have a dominant strategy?

(b) Find all pure-strategy Nash equilibria.

(c) Let $a$ be the probability that Player 1 plays $S$ and $b$ the probability that Player 2 plays $S$. Find the mixed-strategy equilibrium in which both players mix. What is each player's expected payoff?

**Exercise 3 (attitudes to risk)**

(a) Classify each utility as risk averse, neutral or loving: $u = \ln c$, $u = 3c + 2$, $u = c^2$, $u = c^{1/3}$.

(b) Compute $r_A(c)$ and $r_R(c)$ for $u = \ln c$. Does this person dislike a \$10 bet more or less as she gets richer?

(c) Sam has $u = \ln c$ and wealth 64. With probability 1/2 he loses 48. Find his expected wealth, expected utility, certainty equivalent and risk premium. Draw the picture.

(d) What is the most Sam would pay for a contract that fully covers the loss? Compare with the expected loss.

**Exercise 4 (how much insurance?)** Ana has wealth 100 and $u = \ln c$. Her shop burns down with probability $1/4$, a loss of 80. She can buy coverage $\alpha \in [0, 80]$ at price $q$ per unit.

(a) Write final wealth in each state and her maximization problem. Derive the first-order condition.

(b) What premium $q$ lets the insurer break even on average? Show that at this price Ana fully insures.

(c) Now $q = 1/3$. Find the optimal coverage and wealth in each state. Is insurance full, partial or absent?