Econ 101A
Section 1

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1 Preferences and Utility Functions

1.1 Properties of Preferences

- $\succsim$ is **complete** if for all x and y in X either $x \succsim y$, or $y \succsim x$ or both.
- $\succsim$ is **transitive** if for all x,y, and z, $x \succsim y$ and $y \succsim z$ implies $x \succsim z$.
- $\succsim$ is **rational** if $\succsim$ is *complete* and *transitive*.
- A rational preference $\succsim$ is **continuous** if for all x,$y \in X$ such that $x \succ y$, we can find some $\epsilon > 0$ such that for all $x'$and y' that are less than ϵ distant from x and y, respectively, we have $x' \succ y'$.
- $\succsim$ **reflexive** if $x \succsim x$ for every $x \in X$. (there's always a weak preference between a choice and itself).
- $\succsim$ is **monotonic** if $x \geq y$ implies $x \succsim y$ [1]
- $\succsim$ is **strictly monotonic** if $x \geq y$ and $x_j > y_j$ for some j implies $x \succ y$.
- $\succsim$ is **convex** if for all x, y, and z in X such that $x \succsim z$ and $y \succsim z$, then $tx + (1-t)y \succsim z$ for all t in $[0, 1]$.

1.2 Utility Function

Definition: A utility function $u : X \to \mathbb{R}$ represents $\succsim$ if, for every pair of points x and y in X , $x \succsim y$ if and only if $u(x) \geq u(y)$. In other words, if x has higher utility than y , you prefer x to y.

- **Theorem (Existence):** If preference relation $\succsim$ is rational and continuous on set X, there exists a continuous utility function $u : X \to \mathbb{R}$ that represents it.
- **Non-uniqueness:** The utility function representing preferences is not unique. If $u(x)$ represents preferences $\succsim$ and f is a strictly increasing function, then $f(u(x))$ represents $\succsim$ as well. For most of the time we only care about the **ordinal** property of utility function.

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[1] If $x = (x_1, ..., x_n)$ and $y = (y_1, ..., y_n)$ are n-dimensional vectors, $x \geq y$ means that $x_j \geq y_j$ for $j = 1, ..., n$

2 Indifference Curves

Let's stick to the 2-good case (x and y are our goods). When this happens, indifference curves take the following form for some constant c:

$$u(x, y) = c$$
$$\Leftrightarrow u(x, y) - c = 0$$

Take the latter expression, which is set equal to 0. We can treat y as our 1 unknown variable for this single equation and x as a parameter. (In other words, think of this expression as saying $h(y; x) = u(x, y) - c$). By IFT, if the above expression is differentiable and that $\frac{\partial u(x,y)}{\partial y} \neq 0$, then:

- we can write y as a function of x and our constant c
- we can determine $\frac{dy}{dx}$ - note that the negative of this quantity is called the **marginal rate of substitution** (MRS). Specifically, $\frac{dy}{dx} = -\frac{\partial u/\partial x}{\partial u/\partial y}$. You can think of the MRS as how much of y you would need in exchange for x in order to stay at the same utility. Visually, this would be the negative of the slope of an indifference curve.

3 Exercises

3.1 Preferences and Utility Functions

Suppose there are two goods in the world: apples (x) and oranges (y), and that our preferences can be represented by the following utility function: $u(x, y) = 2x + y$

1. If a consumer does not have any apples, how many oranges would he need to be indifferent to a consumption bundle containing 2 apples and 1 orange?
2. Draw the indifference curves for when the utility equals 5 units and when the utility equals 7 units. Do these indifference curves cross? Can they ever cross? Why or why not?
3. Prove that the preferences represented by this utility function are transitive. That is, show that if $(x_1, y_1) \succsim (x_2, y_2)$ and $(x_2, y_2) \succsim (x_3, y_3)$, then $(x_1, y_1) \succsim (x_3, y_3)$.

3.2 Utility function

Consider the following utility function, with $\alpha > 0$ and $\beta > 0$:

$$u(x, y) = \alpha \log(x) + \beta \log(y)$$

Recall the definition of **monotonic preferences** (basically, *more is not worse*).

1. Are the preferences represented by the above utility function monotonic? Explain.
2. What is the sign of the slopes of our indifference curves? Are they upward sloping or downward sloping?
3. Let's consider another utility function, the utility function: $\tilde{u}(x, y) = x^\alpha y^\beta$. Prove that the two utility functions u and $\tilde{u}$ represent the same preferences; that is, prove that

$$u(x_1, y_1) \geq u(x_2, y_2) \Leftrightarrow \tilde{u}(x_1, y_1) \geq \tilde{u}(x_2, y_2)$$

4. Suppose currently market price of x is p, price of y is 1. Currently consumer is consuming at the point (x_0, y_0). What would consumer do if $MRS(x_0, y_0) > p$? What would consumer do if $MRS(x_0, y_0) < p$?
5. Suppose now the utility function has changed to $u(x, y) = 2x + y$. How would your answer in Part (4) differ?