Econ 101A
Section 9

Clotaire Boyer

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1 Time Inconsistency in Intertemporal Choice

1.1 Three Time Periods with Time Consistent Preferences

- U is the utility function. Assume $U' > 0$ and $U'' < 0$.
- The setup is like the two period case we saw before the midterm, just with an additional period.
  - Three periods ($t = 0, 1, 2$) with earnings M_0, M_1, and M_2, respectively. We are trying to choose the optimal consumption path, that is, we are choosing c_0, c_1, and c_2.
  - $M_i' = M_i +$ savings $-$ debts from previous period.
  - Discount rate δ
  - interest rate r
- In this case, here is our optimization problem in period 0:

$$\max_{c_0, c_1, c_2 \geq 0} \; U(c_0) + \frac{1}{1+\delta} U(c_1) + \left(\frac{1}{1+\delta}\right)^2 U(c_2)$$
$$\text{s.t. } \; c_0 + \frac{1}{1+r} c_1 + \left(\frac{1}{1+r}\right)^2 c_2 = M_0 + \frac{1}{1+r} M_1 + \left(\frac{1}{1+r}\right)^2 M_2$$

- Notice that we are discounting the utility in period 2 relative to period 1 by the same amount that we are discounting the utility in period 1 relative to period 0.
- Ratio of FOCs: $\frac{U'(c_1^*)}{U'(c_2^*)} = \frac{1+r}{1+\delta}$
- Now, period 0 is over, the new optimization problem in period 1 is:

$$\max_{c_1, c_2 \geq 0} \; U(c_1) + \left(\frac{1}{1+\delta}\right) U(c_2)$$
$$\text{s.t. } \; c_1 + \left(\frac{1}{1+r}\right) c_2 = M_1' + \left(\frac{1}{1+r}\right) M_2$$

- Ratio of FOCs: $\frac{U'(c_1^*)}{U'(c_2^*)} = \frac{1+r}{1+\delta}$. Same as before.
- Decision made in the past about future consumption **consistent** with decision made when future arrives.

1.2 Three Time Periods with Time Inconsistent Preferences

- The model is the same as above, except we now have an *additional* present-focus parameter $\beta < 1$ that discounts *everything in the future by the same amount.*
- In other words, here is our new maximization problem:

$$\max_{c_0,c_1,c_2\geq 0} \quad U(c_0) + \frac{\beta}{1+\delta}U(c_1) + \frac{\beta}{(1+\delta)^2}U(c_2)$$
$$\text{s.t. } c_0 + \frac{1}{1+r}c_1 + \left(\frac{1}{1+r}\right)^2 c_2 = M_0 + \frac{1}{1+r}M_1 + \left(\frac{1}{1+r}\right)^2 M_2$$

- Ratio of FOCs: $\frac{U'(c_1^*)}{U'(c_2^*)} = \frac{1+r}{1+\delta}$
- Now period 0 is over, the new optimization problem in period 1 is:

$$\max_{c_1,c_2\geq 0} \quad U(c_1) + \left(\frac{\beta}{1+\delta}\right)U(c_2)$$
$$\text{s.t. } c_1 + \left(\frac{1}{1+r}\right)c_2 = M_1' + \left(\frac{1}{1+r}\right)M_2$$

- Ratio of FOCs: $\frac{U'(c_1^*)}{U'(c_2^*)} = \frac{\beta(1+r)}{1+\delta} < \frac{(1+r)}{1+\delta}$ since $\beta < 1$.
- That is, the agent **consumes more in period 1 than what is previously planned in period 0** (since we assume $U'' < 0$).
- Decision made in the past about future consumption **inconsistent** with decision made when future arrives.

1.3 Commitment Device

- Time-consistent agents do not need a commitment device.
- With a commitment device (e.g., maybe a computer program that will enforce period 0 decisions for the future you), a time-inconsistent agent's future consumption path is decided by the optimal decision made in period 0. That is, c_0^*, c_1^*, c_2^* from solving

$$\max_{c_0,c_1,c_2\geq 0} \quad U(c_0) + \frac{1}{1+\delta}U(c_1) + \left(\frac{1}{1+\delta}\right)^2 U(c_2)$$
$$\text{s.t. } c_0 + \frac{1}{1+r}c_1 + \left(\frac{1}{1+r}\right)^2 c_2 = M_0 + \frac{1}{1+r}M_1 + \left(\frac{1}{1+r}\right)^2 M_2$$

- That is, a time-inconsistent agent with a commitment device behaves like a time-consistent agent.
- A time-inconsistent agent can achieve higher overall utility with a commitment device, from time 0's point of view.
- In fact, this implies that the agent at time 0 is willing to pay for commitment devices.

2 Exercise

2.1 Time inconsistency

Amy inherits M dollars when born. She will live for 3 periods (t = 0, 1, and 2) and must allocate M over her lifetime so as to consume a utility maximizing amount of cookies, c_t. She has no other source of income, but she can save each period and earn r for each dollar saved. Her utility function at period 0 is $U_0(c_0; c_1; c_2) = u(c_0) + \frac{\beta}{1+\delta}u(c_1) + \frac{\beta}{(1+\delta)^2}u(c_2)$, where $\beta \in (0, 1)$ and $u(c_t) = (c_t)^{1/2}$.

1. When Amy gets to period 1, how will she allocate her current wealth between cookies in that period (c_1^*) and cookies in the next period (c_2^*)?

2. Suppose that in period 0, Amy can choose her levels of c_1, c_2 in advance, with a commitment device. What would she choose as c_1^{*c}, c_2^{*c}, as fractions of her wealth at t = 1?