ECON 100A - Section Notes
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1 Producer 1 Consumer - the Robinson Cruoe Economy

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1. Setup. The Robinson Crusoe (R–C) economy is the simplest general equilibrium model: a single consumer–producer who both supplies labor and consumes goods. It provides a bridge between micro theory (preferences and production) and general equilibrium analysis.

- One agent: Robinson Crusoe.
- One consumption good x and one input (labor) ℓ.
- Endowment of time: $\bar{L}$, which can be split between leisure L^s and labor $\ell = \bar{L} - L^s$.

2. Preferences.

$$U(x, L^s) \quad \text{with } U_x > 0,\ U_{L^s} > 0.$$

Crusoe values both consumption and leisure.

3. Technology.

$$x = f(\ell), \quad f'(\ell) > 0,\ f''(\ell) < 0.$$

Labor produces output with diminishing returns.

4. Decentralized interpretation. We can think of Crusoe as operating a firm that hires labor from himself at wage w and sells output at price p. The firm problem:

$$\max_{\ell}\ \pi = pf(\ell) - w\ell.$$

The FOC (profit maximization) gives:

$$pf'(\ell^*) = w.$$

Crusoe-as-consumer then chooses consumption and leisure:

$$\max_{x, L^s}\ U(x, L^s) \quad \text{s.t.} \quad px = w(\bar{L} - L^s) + \pi.$$

The FOC (utility maximization) implies:

$$\frac{U_{L^s}}{U_x} = w/p.$$

5. Equilibrium. In equilibrium, Crusoe's choices as a producer and consumer must be consistent:

$$\ell^* = \bar{L} - L^{s*}, \quad x^* = f(\ell^*).$$

The wage w and price p are relative and determined up to a numéraire. The equilibrium satisfies both:

$$pf'(\ell^*) = w \quad \text{and} \quad \frac{U_{L^s}}{U_x} = \frac{w}{p}.$$

Hence,

$$\frac{U_{L^s}}{U_x} = f'(\ell^*),$$

so the marginal rate of substitution (MRS) between leisure and consumption equals the marginal product of labor (MPL).

6. Interpretation.

- The R–C economy illustrates that a competitive equilibrium is Pareto efficient: $MRS = MPL$.
- It unifies consumer and producer theory: the same person optimizes both sides.
- In multi-agent extensions, this logic generalizes to the First Welfare Theorem.

Key takeaway: in equilibrium, Crusoe works until his personal trade-off between leisure and goods equals the economy's technological trade-off between labor and output.

Practice

Problem 1

Masa is an economy of one. As a profit-maximizing producer, he operates *MasaFilm*, which makes movies (y) using a single input, labor (l), with production function:

$$y = 6\sqrt{l}.$$

As a utility-maximizing consumer, he is the sole owner of *MasaFilm* and so receives its profit, and is the sole supplier of labor to *MasaFilm*. He likes watching movies and dislikes working, with utility function:

$$u = 3y - \frac{9}{2}l^2.$$

Let the price of movies be normalized to 1, and let the wage rate be w.

1. Set up and solve *MasaFilm*'s profit maximization problem to find its optimal output y_S^*, its optimal labor demand l_D^*, and its profit π, all as a function of w. In a sentence or two, explain in simple terms why it would be worse for *MasaFilm* to produce a little bit more than y_S^*.
2. Set up and solve Masa the consumer's utility maximization problem to find his optimal demand for movies and his optimal supply of labor as a function of w. Then find and write down a competitive equilibrium in this economy.