ECON 100A - Section Notes
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We study how to aggregate individual preferences into a *social preference ordering*. Let each agent $i \in \{1, \ldots, n\}$ have utility $u_i(a)$ over allocation a. A *social welfare function* (SWF) maps the vector of individual utilities into a scalar measure of collective welfare:

$$W(a) = f(u_1(a), \ldots, u_n(a)),$$

with f increasing in each argument. Examples include:

$$\text{Utilitarian: } W_U(a) = \sum_i u_i(a), \qquad \text{Rawlsian (minimax): } W_M(a) = \min_i u_i(a),$$

$$\text{Cobb–Douglas: } W_C(a) = \prod_i u_i(a).$$

Each implies different ethical weights on inequality and efficiency. If allocation a^* maximizes $W(a)$ over feasible allocations, then a^* is Pareto efficient. Conversely, every Pareto efficient allocation can be justified as welfare-maximizing under some W. When markets exist for all payoff-relevant goods (no externalities), the First and Second Welfare Theorems ensure that any competitive equilibrium allocation is Pareto efficient and hence welfare-maximizing for some SWF.

Practice

Problem 1

Consider an economy with three people, A, B, and C. There is a public good that costs \$300 in total to provide. A would be willing to pay \$120 for the good, B would be willing to pay \$90\$, and C would be willing to pay \$X\$.
If the public good is provided, all three will pay an equal share of the total cost. Each person will be asked to report their net valuation for the good. If the reports sum to more than zero, the good will be provided; otherwise, it will not be provided.
Pivotal people, if any, in the reporting process must pay a *Clarke tax* equal to the absolute value of the sum of all other agents' reported net valuations. Each person's total payoff is their net valuation minus any Clarke tax if the good is provided, and zero minus any Clarke tax if the good is not provided.

- If everyone reported their true net valuation, for what values of X is the good provided or not? Is the outcome socially efficient when everyone reports truthfully? With reference to the definition of a public good, why?
- If everyone reported their true net valuation, B is pivotal and pays \$5 Clarke tax. What is X? Who else is pivotal and how much Clarke tax do they owe? Would the pivotal people have been better off reporting a false net valuation? Why or why not?

- Say that B and C reported their true net valuations. If there had been no Clarke tax, could A do better by reporting falsely rather than truthfully? With the Clarke tax, would the answer to that be the same or different? Explain why.

Problem 2

There are two people, Mo and Tori, and two goods, fruit f and candy c. Mo's utility function is $u_M = f_M + 2c_M$ and Tori's is $u_T = 2f_T + c_T$, where f_M is the amount of fruit that Mo has. There are 6 total units of each good in the economy.

Find two allocations of goods such that allocation 1 is "better" than allocation 2 under each of the utilitarian, minimax, and Cobb–Douglas social welfare functions, but allocation 2 is Pareto efficient while allocation 1 is not. No written explanation is required, but clearly show all relevant calculations.