ECON 100A - Section Notes
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Announcements

- Essay Planning Due in 2 days

Basics on Game Theory

Normal (strategic) form. A finite normal-form game is a tuple

$$G = (N,\, (S_i)_{i\in N},\, (u_i)_{i\in N}),$$

where

- $N = \{1, \ldots, n\}$ is the set of players;
- S_i is the (finite) set of pure strategies of player i;
- $u_i : \prod_{j\in N} S_j \to R$ is player i's payoff function.

We often represent G by a *payoff matrix* (table) when there are two players and few strategies.

Best responses and Nash equilibrium. Given a strategy profile $s = (s_i)_{i\in N}$, write s_{-i} for the strategies of all players except i. A *best response* of player i to s_{-i} is any $s_i \in S_i$ such that

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \quad \forall s_i' \in S_i.$$

A (pure-strategy) *Nash equilibrium* is a profile s^* such that for all i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*) \quad \forall s_i' \in S_i,$$

i.e. each player's strategy is a best response to the others' strategies.

Extensive form (game trees). An extensive-form game specifies:

- a game tree with a distinguished initial node and terminal nodes (outcomes);
- for each decision node, which player moves there and what actions are available;
- *information sets*, grouping decision nodes that a player cannot distinguish when moving;
- payoffs at each terminal node.

A (pure) *strategy* for player i in the extensive form is a complete contingent plan: an action specified at every information set of player i.

Simultaneous vs. sequential moves.

- In simultaneous-move games (e.g. standard payoff matrix), players choose actions without observing others' current choices. In extensive form this is modeled by an information set linking their decision nodes.
- In sequential-move games with perfect information, players move in a known order and observe all previous actions. Each decision node is then a singleton information set.

The same situation can often be described both in extensive form (tree) and in normal form (table); the *normal form* is obtained by listing all strategies in the tree and the induced payoffs.

Subgames and SPNE. A *subgame* of an extensive-form game (with perfect information) is a subtree that:

1. starts at a single decision node that is not in an information set with any other node;
2. if a node is included, all its successors are included;
3. includes entire information sets (no partial info sets).

A strategy profile is a *subgame perfect Nash equilibrium* (SPNE) if it induces a Nash equilibrium in every subgame. SPNE rules out non-credible threats.

Backward induction. In a finite extensive-form game with perfect information, SPNE can be found by backward induction:

1. Start from the last decision nodes and choose, for each such node, the action that maximizes the moving player's payoff.
2. Replace each solved part of the tree with the resulting payoffs and move one step earlier in the tree.
3. Continue until the initial node is reached.

The strategy profile that corresponds to these choices at every node is a (typically unique) SPNE.

Repeated games. Let G be a stage game with action sets $(A_i)_{i\in N}$ and payoffs $(u_i)_{i\in N}$.

- In a T-period repeated game, G is played $t = 1, \ldots, T$. At each period t, players may observe the history of past actions h^{t-1} and then choose current actions a^t.
- A (pure) strategy for player i is a mapping from histories to actions: $\sigma_i : \mathcal{H} \to A_i$, where $\mathcal{H}$ is the set of all possible histories.

- Payoffs are often the sum or discounted sum of stage payoffs, e.g. for discount factor $\delta \in (0,1)$:

$$U_i = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_i(a^t).$$

Nash equilibrium and SPNE are defined as before, but now using strategies that condition on histories. In repeated games, SPNE again rules out non-credible threats at any history (subgame).

Section Exercises

Take $\sim$ 15 minutes to work on these exercises individually, then turn to your neighbors and discuss your responses in small groups for a few minutes. We will then come together to work through them as a class, with groups sharing their progress/responses.

1. Consider a two player, simultaneous move game. The strategies available to each player and payoffs are shown in the following matrix form:

		Player 2		
		L	C	R
Player 1	T	3, 2	4, 0	2, 1
	M	2, 5	1, 1	1, 3
	B	3, 3	2, 6	0, 2

Game 5

   (a) This game has a unique Nash equilibrium in pure strategies. What is it? Explain fully and in simple terms why its a Nash equilibrium. Is the outcome in that equilibrium Pareto efficient? Why or why not?

   (b) Which strategy profile would result in the outcome that is best under a utilitarian social welfare function? What about under a minimax social welfare function? In each case, explain why its best under that social welfare function.

Discussion Prompts

Break into groups of 3 (different from your section exercise groups). Discuss the prompt for ~ 5 minutes and prepare a (written) summary of your discussion to share with the class. We will then come together and discuss both prompts.

1. Which is more of an obstacle to good game theoretic modeling: the fact that strategy spaces are really complicated in the real world, or the fact that preferences are really complicated in the real world?