# ECON 100A - Section Notes
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## Repeated games with discounting

A (discrete-time) infinitely repeated game is built from a *stage game* $G$ that is played in each period $t = 0, 1, 2, \dots$. In each period, players simultaneously choose actions $a_i^t$ from the stage-game action sets, observe the outcome, then move to the next period.

A *history* at time $t$ is $h^t = (a^0, \dots, a^{t-1})$, where $a^s$ is the action profile in period $s$. A (pure) strategy for player $i$ is a function $s_i$ that assigns an action to every possible history: $s_i : \{h^t\} \to A_i$.

Let $u_i(a^t)$ be player $i$'s stage-game payoff in period $t$. With common discount factor $\delta \in (0, 1)$, the normalized discounted payoff is

$$U_i((a^t)_{t\geq 0}) = (1-\delta) \sum_{t=0}^{\infty} \delta^t u_i(a^t).$$

A *Nash equilibrium* (NE) of the repeated game is a strategy profile $(s_1, \dots, s_n)$ such that no player can gain by deviating unilaterally. A *subgame perfect equilibrium* (SPE) is a NE in every subgame (i.e. after every history $h^t$), so strategies must be credible.

Example: In an infinitely repeated Prisoners' Dilemma, *grim trigger* strategies (cooperate until someone defects, then defect forever) can form an SPE if the discount factor is high enough:

$$\delta \geq \frac{T-R}{T-P},$$

where $T$ is the temptation payoff, $R$ the mutual cooperation payoff, and $P$ the mutual defection payoff. Intuition: cooperation is sustainable only if the threat of future punishment outweighs the short-run gain from deviating.

## Cournot and Bertrand competition

**Cournot (quantity) competition.** There are $n$ firms producing a homogeneous good. Inverse demand is $P(Q) = a - bQ$ with $Q = \sum_{i=1}^{n} q_i$, and constant marginal cost $c$. Each firm simultaneously chooses a quantity $q_i \geq 0$ to maximize profit

$$\pi_i(q) = (P(Q) - c)\, q_i.$$

The Cournot Nash equilibrium with symmetric firms is

$$q_i^* = \frac{a-c}{b(n+1)}, \quad Q^* = \frac{n(a-c)}{b(n+1)}, \quad P^* = a - bQ^* = \frac{a+nc}{n+1}.$$

As $n$ increases, $P^*$ falls and converges to marginal cost $c$ (the competitive outcome).

**Bertrand (price) competition.** Two firms produce a homogeneous good with constant marginal cost $c$. Each firm simultaneously chooses a price $p_i$. Consumers buy from the firm with the lowest price; if $p_1 = p_2$, they split the market. In any Nash equilibrium, we must have

$$p_1^* = p_2^* = c.$$

If either firm tried to set $p > c$, the other could profitably undercut slightly. At $p = c$, any undercutting yields no profit, so the unique NE has $p = c$ and zero economic profit (*Bertrand paradox*).

Cournot and Bertrand show how the *choice variable* (quantity vs. price) and the *timing/information* of moves crucially shape market outcomes, and repeated interaction can help sustain more cooperative (higher-price or lower-quantity) outcomes if firms are sufficiently patient ($\delta$ large).

## Section Exercises

Take $\sim$ 15 minutes to work on these exercises individually, then turn to your neighbors and discuss your responses in small groups for a few minutes. We will then come together to work through them as a class, with groups sharing their progress/responses.

1. Consider an infinitely repeated game. Jims Shady Mechanics is a car repair shop in a small town. Every day, one randomly chosen car in the town breaks down and its owner must decide whether to take it to Jim. If they do, Jim must decide whether to be Honest and fix the car for a fair price, or Cheat the car owner, charging more than is really needed. Jim earns \$100 if he is Honest and \$500 if he Cheats. Lets say that Jims utility each day is equal to the money he earns and that he discounts future payoffs according to the discounted utility model, with $0 < \delta < 1$.

   (a) Assume that people will always take their cars to Jim unless hes ever Cheated anyone. If Jim ever plays Cheat, the car owner immediately knows they have been Cheated, tells everyone in town, and no-one ever goes to Jims Shady Mechanics again. For what values of $\delta$ will Jim always be Honest? Explain in simple terms the intuition for your answer.

   (b) For each of the following cases, briefly discuss how and why it might change the threshold $\delta$ required for Jim to be Honest: (i) theres a chance that a car owner who's been Cheated might not know they've been Cheated and so no-one finds out; (ii) theres a chance that Jim may face costly legal action after he Cheats someone.

2. Profit-motivated duopolists produce an identical product. They will decide sequentially on quantity: firm 1 will pick $y_1$, then firm 2 will observe $y_1$ and pick $y_2$. Demand is given by $p = 17 - \frac{1}{2}y$ (y is total quantity produced by both firms). Each firms cost function is $c(y_i) = y_i$.

   (a) Firm 2's reaction function is $y_2(y_1) = 16 - \frac{1}{2}y_1$. In simple terms (no math) explain (i) what a reaction function is and (ii) how this reaction function can be found in this case.

(b) What is the price of the good in the unique subgame perfect Nash equilibrium of this game? What total quantity is sold? Show all calculations. No explanations required.

(c) If this market had not been a duopoly but instead been served by a monopolist (again with cost function $c(y) = y$, what would the price and quantity sold have been? Based on this answer and your answer to b) (but with no further calculations required): would consumer surplus be higher in the monopoly case or the duopoly case? Explain the simple intuition for how you know.

## Discussion Prompts

Break into groups of 3 (different from your section exercise groups). Discuss the prompt for $\sim$ 5 minutes and prepare a (written) summary of your discussion to share with the class. We will then come together and discuss both prompts.

1. Compare and contrast the Cournot and Bertrand models and their predictions. Which one is the right way to think about a duopoly?